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STEM IIA

Euclidean vs. Spherical vs. Hyperbolic Geometry

How does one know his or her basic geometry theories? Because of a man named Euclid, who lived in Alexandria, Egypt during 300 BC. Euclid was a Greek mathematician, but he can also be known as “The Father of Geometry”. He wrote the Elements which contained thirteen books. The first six books included basic planar or flat geometry. The next four books included the Number Theory. Lastly, the next three books contained solid geometry. People felt the need to search for Spherical geometry because they needed to understand how the world worked. The world is a sphere, therefore making the search for spherical geometry a necessity. Spherical geometry is the study of spheres, and the geometry of figures on the surface of a sphere. The figures will always add up to more than 180 degrees because they are placed on the sphere. Hyperbolic geometry is a form of non- Euclidean geometry that substitutes the parallel postulate with two lines that can be drawn parallel to a line though any point that is not on the line given. In all three types of two dimensional geometry, the definitions, polygons, triangles, lines and circles are imperative.

In Euclidean geometry, the definitions that define what each type of geometry looks for are important because in Euclid’s case, he formed geometry and mathematics by centering all he knew on “common notions” and “postulates” but especially by definitions. There were five postulates that were the basis of Euclidean geometry. First, he explained that a straight line segment can be drawn by joining any two points. Secondly, Euclid said that any straight line segment can be extended indefinitely in a straight line. He then said that if given any straight line segment, a circle can be drawn by having the segment as the radius and one endpoint as the center of the circle. Penultimate, he explained that all right angles are congruent. Lastly, Euclid explicated that if two lines are drawn which intersect a third line, in a way that the sum of the inner angles on one side less than two right angles, then the two lines unavoidably must intersect each other on that side if it is extended far enough. This is known as the parallel postulate. Polygons in Euclidean geometry can have three or more sides. Triangles have several characteristics in Euclidean geometry. Given any line segment, one can create an equilateral triangle, one of the sides is the given line segment. Secondly, if two sides of the included angle of one triangle are equal to two sides and the included angles of another angle, the two triangles are equal. A triangle can have a maximum of one obtuse or right angle. Lastly, the angles opposite to the two equal sides of an isosceles triangle are equal. Circles in Euclidean geometry do not have a maximum magnitude/size. It has a line this is equidistant from a center point of the circle.

In Spherical geometry definitions are important because when explaining Spherical geometry, it gets very enigmatic. All the numbers change when make a triangle on a sphere. Polygons in Spherical geometry can have two or more sides because they are shapes being made on a sphere. For example, a lune can be drawn on a sphere. A lune is a polygon with only two sides. Triangles have several characteristics in Spherical geometry. As the sum of the angles of the triangles change, the area of the spherical triangle changes too. A triangle can have more than one obtuse angle. The sum of angles is 180/540 degrees. Additionally, it is impossible for one to have a similar triangle on a sphere. Lastly, are the circles in Spherical geometry. The largest is the great circle, as the radius of the great circle gets larger, the area becomes smaller and two congruent circles can be made.

In Hyperbolic geometry, the definition is extremely important because it contains some of the rules in Euclidean geometry, but is categorized in non- Euclidean geometry. There are many questions when dealing with Hyperbolic geometry. For example, what is a line? Can there be parallel lines? Can the angle sum of the triangle be greater than 180°? Etc. fortunately; we are able to answer these questions. A line can be made by making two points. There can be more than one parallel line because when drawn on a circle, it creates more lines. The angle sum of a triangle can be less than 180 degrees because when flattened it gets smaller. There can be circles in Hyperbolic geometry and the circles can have any radius.

The three most important two-dimensional geometries contain vital information about their definitions and why they are important, polygons, triangles and their different characteristics, lines and circles. Euclid began this discovery of geometries when he discovered the five postulates, and created Euclidean geometry.