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Comparing and Contrasting Three Types of Geometry

The majority of people hear the word geometry and think of two- dimensional geometry, the kind taught first in schools, and the kind that is easily drawn on paper. This kind of geometry is called Euclidian geometry, but contrary to what a lot of people think, it is not the only kind. Spherical geometry and hyperbolic geometry are two other kinds of geometry that are similar to Euclidian geometry in some ways, but these three types of geometry also are different in many ways. Euclidian geometry was named after Euclid, and he used five postulates to prove Euclidian Geometry. The fifth is the parallel lines postulate, but this postulate does not work in the same way for each of the three geometries and that is why these three geometries differ greatly in some ways. Spherical geometry is two-dimensional geometry, but on a sphere, which causes it to have different characteristics. Hyperbolic geometry was discovered in 1800, about 70 years after Giovanni Girolano wrote a book defending Euclid. These three geometries’ main similarities and differences pertain to the parallel lines postulate, triangles, polygons, and the idea of a straight line.

The parallel lines postulate, which was created by Euclid, states that for any point not on a line, there is exactly one line parallel to the given line. This is true for Euclidian geometry. However, it is not true for hyperbolic or spherical geometry. In spherical geometry, there are no parallel lines. Each line on a sphere is called a great circle, and no lines on a sphere miss each other, so there are no parallel lines. The parallel lines postulate is also not true for hyperbolic geometry. This is because for every line and point that is not on the line, there are infinite amount of lines that go through the point that are parallel to the line. Hyperbolic geometry has more of a three-dimensional shape, as it looks like a piece of kale or a coral in the ocean. The best way to model it however, is through crotchet. When one looks at a model of hyperbolic geometry, it is apparent that there could be an infinite amount of parallel lines to any certain line because it can bend and fold in many ways to make numerous lines. Therefore, the parallel postulate fails for hyperbolic geometry and spherical geometry.

Furthermore, while the sum of the angles in a triangle always adds up to 180 degrees in Euclidian geometry, it is not the same for the other two types of geometry. In spherical geometry, the angle sum for a triangle is more than 180degrees. Each side of the triangle is a part of a great circle. It is possible to find the area of a triangle by adding up the angle measures and subtracting 180 degrees to find the “excess”. The excess is how much greater than 180 degrees the sum is. The excess divided by 720 is the portion of the surface area of the sphere that the triangle takes. Then the sum of all of the lunes equals the area of the sphere. Therefore the area of a triangle = area of the sphere X (< a + < b +< c – 1800 )/720. Hyperbolic geometry is similar to spherical geometry in that the sum of the angles for any triangle varies. However, the angle sum is less than 180 degrees, which is the opposite of spherical geometry. There are no similar triangles that are not congruent and the Pythagorean Theorem is false for hyperbolic geometry. There can be equilateral triangles, and it is possible to make a right angle. Also, the smaller the triangle, the closer the angle sum is to 180 degrees because the triangle is closer to being on a flat surface. All in all, Euclidian, Spherical, and hyperbolic geometry share some characteristics in terms of triangles, but they are also different in some ways, such as their angle sums.

Finally, polygons differ for each of the three different types of geometry. In Euclidian geometry, a polygon has three or more sides. They can range from triangles to octagons, to shapes with twenty different sides. In spherical geometry, polygon does not have to have three sides, it can only have two. These two sides are made of lunes, which are shapes that go from one point to another point, and slices a piece of the sphere. This is one major difference between spherical and Euclidian geometry; the number of sides required to make a polygon is not the same. In hyperbolic geometry, a polygon requires three sides, just as it does in Euclidian Geometry. However, it is not possible to have a rectangle in hyperbolic geometry, which is different from Euclidian geometry. However, it is possible to make a square, meaning a polygon where each side is the same length and each angle is the same measure.

To conclude, Euclydian geometry, spherical geometry, and hyperbolic geometry resemble each other in many ways, but they also have unique characteristics that set them apart from one another. The parallel lines postulate works differently for each one, but since it fails for both hyperbolic and spherical geometry, one might conclude that they are the closest of the three geometries. However, in terms of triangles, the three different geometries vary greatly and it does not seem that any of them are similar. When looking at polygons for each type of geometry, it can be concluded that hyperbolic and Euclidian geometry are the closes because they both require polygons with three sides. Overall, each of the three types of geometry is different in its own way. It is interesting to see how geometry can differ so much depending on what type of geometry it is. Also, it is intriguing to think about all of the differences they have, and then realize that there are still many ways in which hyperbolic geometry, spherical geometry, and Euclidian geometry are related.