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Geometries

Geometry has been used in the past to make revolutionary discoveries and is still used every day to help solve less significant world problems. Even though two dimensional geometry is generally the most well-known, there are actually three types of common geometries: Euclidean, spherical, and hyperbolic. Euclidean geometry is based on flat surfaces, while spherical geometry is based on spheres, and hyperbolic geometry is based on non-flat surfaces. Furthermore, Euclidean geometry has 5 main postulates which state that there is a straight line from point to any point, a finite straight line can be produced in any straight line, there is a circle with any center and any radius, all right angles are equal to one another, and if two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough (Parallel Postulate). Although these geometries have many some similarities, they also have many differences such as when dealing with parallel lines and polygons.

There are many similarities and differences between spherical and Euclidean geometry. In both geometries, line segments are possible where any two points on the surface can be connected by a straight path. A line in spherical geometry is along the great circle, the path with the least distance. However when dealing with lines, parallel lines are possible in Euclidean geometry, but not in spherical geometry because lines in spherical geometry are only along the great circle and therefore they always intersect, and there are no parallel lines. In addition, in Euclidean geometry, the angle sum of a triangle is always 180 degrees, however, in spherical geometry, the angle sum of a triangle can be anywhere between 180 degrees and 540 degrees, but not quite 180 or 540 degrees. For the angle sum of a triangle to be close to 180 degrees in spherical geometry, the triangle must be very small so it is as if the spherical surface is flat. Similar triangles are also not possible on a spherical geometry while they are possible in Euclidean geometry. Lastly, Euclidean geometry requires a polygon to have at least three sides; whereas in spherical geometry, a polygon only has to have two sides. This two sided polygon in spherical geometry is called a Lune and has two vertices on opposite sides of the sphere along the great circle. Therefore, spherical and Euclidean geometry have some similarities when dealing with lines and triangles, but they also have differences in parallel lines and triangles.

In addition to spherical and Euclidean geometries, spherical and hyperbolic geometries also have many similarities and differences. As stated before, triangles in spherical geometry can have an angle sum from 180 degrees to 540 degrees without actually reaching those exact measurements. On the contrary, in hyperbolic geometry, triangles can have an angle sum anywhere from 0 degrees to 180 degrees without actually reaching those magnitudes. Therefore they are similar in this way because they both cannot reach 180 degrees, but triangle angle sums are always significantly smaller in hyperbolic geometry than in spherical geometry because in spherical geometry, the surface is curved outwards, enlarging angle measures; whereas in hyperbolic geometry, the surface is curved inwards, reducing angle measurements. They are also similar because in both geometries, equilateral triangles are possible and the angles in the equilateral triangles are equal, but the angle measures change as the length of the sides change. In addition, while parallel lines are not possible in spherical geometry, there are infinite parallel lines for each line segment in hyperbolic geometry because of the many curves on a hyperbolic surface. Even though, line segments are possible in both geometries. Lastly, when dealing with polygons, hyperbolic geometry requires them to have at least three sides; whereas they only need two sides in spherical geometry. In addition, squares on a hyperbolic surface will always have lower angle sums than those on a spherical surface because of their opposite curves. Therefore spherical and hyperbolic geometry have similarities concerning line segments and equilateral triangles, but differ when concerning triangle angle sums, parallel lines, and polygons.

Finally, hyperbolic and Euclidean geometries also have many similarities and differences. In both geometries, line segments are possible; however, in Euclidean geometry there is only one parallel line for every line where in hyperbolic geometry, there are an infinite number of parallel lines for any given line segment. In addition, a triangle’s angle sum in Euclidean geometry is always 180 degrees, where in hyperbolic geometry, a triangle angle sum can be anywhere from 0 to 180 degrees, but it cannot actually reach those measurements. There are also similar because both geometries require their polygons to have at least three sides. However, the angle measures in hyperbolic geometry are significantly less than angle sums of the same conditions in Euclidean geometry because hyperbolic surfaces curve inwards. Lastly, squares in Euclidean geometry all have angles of 90 degrees. In hyperbolic geometry, squares are possible and all four angle measures would be the same, but their angles would be less than 90 degrees. In these ways, hyperbolic and Euclidean geometry are similar when dealing with line segments and the number of sides their polygons require, but they differ when concerning parallel lines and angle measures.

Over all, all three geometries are very diverse, but still attain some similar aspects. They differ completely with parallel lines and triangle angle sums, but follow identical rules when considering line segments. In the real world, spherical geometry can be used when planning boat or airplane trips. It is also used when handling sphere shaped real objects, like balls or globes. Euclidean geometry is used to draw conclusions about flat surfaces. Therefore this geometry would be used frequently in everyday life because since people live on such a small part of a huge sphere, earth, surfaces seem flat to them. Lastly, Hyperbolic geometries are helpful in real life when dealing with aspects of space and the larger sense of the universe.