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Comparing Hyperbolic, Euclidean and Spherical Geometry

Spherical geometry, hyperbolic geometry, and Euclidean geometry have similar concepts, but the curvature of the surface in spherical geometry and in hyperbolic geometry causes difference between Euclidean Hyperbolic and Spherical geometry. Euclidean geometry is the geometry of an infinite flat one-dimensional plane. Hyperbolic geometry is the geometry of an infinite plane with curvature. Spherical geometry is geometry of the one-dimensional surface of a sphere. Spherical Hyperbolic and Euclidean geometry can be compared and contrasted by examining lines and points, the angles of a triangle and rectangle, polygons, circle, the parallel postulate and daily applications.

In spherical geometry and hyperbolic geometry, a geodesic is a curve which uses the shortest distance to connect two points. In Euclidean geometry a line can be compared to a geodesic. A line is the shortest way to connect two points on a plane. A geodesic is the shortest way to connect two points on a sphere or hyperbolic plane. The first and second Euclidean postulates (“There is a straight line/geodesic from any point to any point” and “A finite straight line can be produced in any straight line.”) apply to both spherical geometry hyperbolic geometry and Euclidean geometry if the word geodesic is applied when talking about spherical and hyperbolic geometry. When talking about three points on a plane, one point will always be in between the other two. In hyperbolic geometry, when given three points on a hyperbolic plane, one point will always be in between the other two. Yet, on a sphere when given three points any point can be in between the other two on a great circle because there is not a beginning or an end.

Circles can be created in Euclidean, Spherical and Hyperbolic geometry, therefore the third postulate (“Given any straight lines segment, a circle can be drawn having the segment as radius and one endpoint as center”) works for all the geometries. In both Euclidean and hyperbolic geometry, as the radius increases so does the area. In Euclidean geometry, the ratio of radius to area is found using the priorities of pi. In hyperbolic geometry it is not known if there is a radius to area ratio to the inconsistent changing of the hyperbolic plane. The radius to area ratio of a circle differs in spherical geometry. Since a sphere is not an infinite plane, the area of a circle increases as the radius increases until it covers half the area of the circle. Once it covers more than half of the circle’s area as the radius gets larger the area gets bigger.

In Euclidean geometry, the sum of a triangle’s angles are 180 degrees. In spherical geometry, the sum of triangle’s angles range from above 180 degrees to below 540 degrees. In hyperbolic geometry, the sum of the triangle’s angles are less than 180 degrees. The angles of a triangle on a sphere or hyperbolic plane change in relation to the area of the triangle. The more area a triangle covers, the larger the sum of the angles will be. Also in Euclidean geometry, all angles in a square are equal to 90 degrees. In spherical and hyperbolic geometry, a sum of 90 degree cannot be formed. Squares only exist in Euclidean geometry. The fourth postulate (“all right angles are equal to one another”) applies to all three, but in spherical and hyperbolic geometry a quadrilateral cannot be formed with four right angles because a triangle’s angle sum cannot be 180 degrees.

In both spherical hyperbolic and Euclidean geometry polygons can be formed. In Euclidean geometry, a polygon most have three of more sides. Yet, in spherical geometry a polygon can have two sides. The two sided polygon is called a lune. A lune is formed by two great circles. In hyperbolic geometry is not known if a polygon can be made with two sides, but it is known for certain a polygon can have three or more sides.

The fifth Euclidean Postulate (“If a straight line falling on two straight lines makes an interior angles on the same side less than two right angles, the two straight line, if produced indefinitely, meet on that side on which the angles are less than the two right angles.”) is called the parallel line postulate. In Euclidean geometry for any point not on a line, there is exactly one line parallel to the line given. The postulate does not apply in spherical geometry and hyperbolic geometry. In spherical geometry, for any point not on a line, there are zero lines parallel to the line given. That is because a sphere has great circles not lines. The great circle continuously wraps around the sphere. Due to a sphere not having an infinite amount of space, the great circles must cross. In hyperbolic geometry, for any point on a line, there is an infinite number of lines parallel to the given line. That is because in hyperbolic geometry the plane has a curvature. The curvature changes throughout the line causing lines to never intersect.

Euclidean geometry is applied when building. Construction is modeled off Euclidean geometry. Architects uses parallel lines in Euclidean geometry and 360 degree quadrilaterals to build flat and symmetrical buildings. Euclidean geometry is applied in the local world surrounding us. People live in a Euclidean world. The world itself is spherical therefore spherical geometry applies when traveling across vast sections of the earth. When flying on an airplane the shortest distance to travel is a geodesic on a sphere. When underwater in the ocean geodesics are also the shortest way to travel. When sea life communicates through echolocation, the echoes travel in a geodesic. The greater universe though is believed to be a hyperbolic plane. Though it is not proven yet due to its fast rate of expansion and wide range of parallels. Hyperbolic geometry can also be applied to coral reefs. Their different surface textures are created because there is more than 360 degrees around a point. People live in a Euclidean society, in a spherical world, in a hyperbolic universe.

Hyperbolic, Spherical and Euclidean geometry’s concepts are comparable, but the Five Euclidean Postulates do not apply to spherical and hyperbolic geometry. Due to the curvature of a sphere and hyperbolic plane, traditional rules of Euclidean geometry do not apply to spherical and hyperbolic geometry.