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STEM: Euclidean Geometry and Spherical Geometry

Alexandria Euclid was a Greek mathematician who wrote *The Elements*, which was composed of thirteen books. In his works of literature, Euclid gave a detailed explanation of geometry and mathematics by constructing definitions, common notions, and postulates about points, straight lines, and angles. The formation of Euclidean Geometry was established directly from Euclid’s books and his five famous postulates.

On the other hand, spherical geometry is a non-Euclidean geometry that studies the two- dimensional surface of a sphere through geodesics and great circles. A geodesic is a curve that traces the shortest distance between any two points, and a great circle is circle on a sphere cut out by intersecting the sphere with a plane through its center. Albert Girard was a French mathematician who established an important formula and theorem in spherical geometry, known as Girard’s theorem and Girard’s formula.

Euclid proved the legitimacy of Euclidean Geometry predominantly through his five postulates: 1. There is a straight line from any point to any point. 2. A finite straight line can be produced in any straight line. 3. There is a circle with any center and any radius. 4. All right angles are equal to one another. 5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles. However, in spherical geometry, there are no straight lines. If you replace “straight line” with “geodesic” in Euclid’s postulates, the fifth postulate, known as the parallel postulate, is contradictory to spherical geometry. Two great circles will always intersect in two points on a sphere; therefore, it is impossible to have parallel lines on a sphere.

Additionally, in Euclidean geometry, a three-sided polygon (triangle) is the shape that has the least amount of sides. While in spherical geometry, two-sided polygons exist and are referred to as lunes. Triangles are also contradictory in Euclidean and spherical geometry. In Euclidean geometry, the sum of all the angles in a triangle are equal to 180 degrees, but in spherical geometry, the sum of all the angles in a triangle are greater than 180 degrees and less than 540 degrees. The sum of the angles on a spherical triangle must be greater than 180 degrees and less than 540 degrees because it is impossible to have a straight line on a sphere, so the sides of the triangle are actually geodesics. Girard’s theorem was derived from the study of triangles on spheres. In his formula, area of the triangle= e/720 × area of the sphere, he states that the area of the sphere is directly proportional to the excess area sum of the angles. In order to find the area of a sphere, area of the triangle, and the excess area sum of the angles, each sides of the triangle must be extended into a great circle. The overlapping great circles form lunes, or two-sided polygons. Because the sum of the angles on a triangle is always greater than 180 degrees, it is also impossible to draw a rectangle, by definition, in spherical geometry.

Euclidean geometry and spherical geometry were mathematically proven to be separate subsections of geometry, even though they share some similar rules.