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Euclidean geometry vs. Spherical geometry vs. Hyperbolic geometry

There are four major aspects that differ in spherical geometry, hyperbolic geometry, and Euclidean geometry. The first aspect is that the meaning of a line and line segment. In Euclidean geometry, a line is a band that has an infinite length. In spherical geometry, a line is called a great circle. A great circle is a circle around a sphere that intersects the “equator” of the sphere. In Euclidean geometry, a line segment is a finite line between two points. The equivalent to a line segment in spherical geometry is a geodesic. A geodesic is a curve on a sphere that goes from one point to another point in the shortest distance. In hyperbolic geometry, lines appear to be geodesic but at the same time there are no great circles. Another difference with lines between the three types of geometry is that there are infinite lines in both Euclidean and hyperbolic geometry but in spherical geometry there are only finite lines. Definitions are important because they changed depending on what type of plane one is on whether it is Euclidean, spherical, or hyperbolic.On a flat surface, in Euclidean geometry, there are lines and line segments, in spherical geometry, they become great circles and geodesics, and in hyperbolic geometry there are geodesics and lines.

Another difference between Euclidean and spherical geometry is the types of polygons that can be formed on a plane or a curved surface. In Euclidean geometry, the smallest shape that can be made is a three sided shape, such as the triangle. On a plane it is impossible to draw a two sided shape because the lines will completely overlap or not connect at all, which doesn’t form a shape. In spherical geometry, the smallest polygon that can be formed is a two sided polygon. The two sided polygon is called a lune. A lune is formed by two great circles. A two sided shape is possible on a sphere because the lines are curved rather than straight. In both cases, for Euclidean and spherical geometry, the number of sides on a polygon can be unlimited.We do not know if there are two sided polygons in hyperbolic geometry because it is beyond our knowledge. We do know that there can be polygons with three or more sides. The difference with polygons between the types of geometry is that it is possible to have a two sided shape, in spherical geometry, but not in Euclidean geometry, while it is unknown to us in hyperbolic geometry.

The third difference between Euclidean and spherical geometry is the sum of the angles in a triangle. In Euclidean geometry, the sum of that angles in a triangle is 180 degrees. However in spherical geometry, the sum of angles in a triangle has to be more than 180 degrees. On a sphere, when you draw a triangle, with an angle sum of 180 degrees it is very small, and when the triangle is that small it is as if it was on a flat surface on the sphere. When you draw a larger triangle, it covers more of the sphere making the sum of the angles more than 180 degrees. In hyperbolic geometry, the sum of the angles in a triangle is less than 180 degrees. When drawing a large triangle, in hyperbolic space, the angles of the triangle become pinched which makes each angles less than 60 degrees. When you draw a smaller triangle, the angle sum can by very close to 180 degrees. Furthermore, this means that the Pythagorean Theorem does not work for spherical and hyperbolic geometry.

The last difference between Euclidean geometry and spherical geometry is the parallel postulate. Euclid wrote the postulates for geometry on a flat surface or a plane. The fifth postulate or the parallel postulate says that if there are two straight lines and one line crosses them and the interior angles on the same side of both lines if less than 90 degrees then they will meet, so they are not parallel. The fifth postulate is true for Euclidean geometry but not for spherical geometry. In spherical geometry, the two lines would be great circles and great circles intersect at two places. This means that a given line through a given point has no parallel lines. The rest of the postulates work for spherical geometry but every time it says line replace it with great circle and when it says line segment replace it with geodesic. In hyperbolic geometry, the parallel postulate also does not work. There can be an infinites number of lines that can be parallel to a certain point. Therefore, the parallel postulate only works for Euclidean geometry.

Applications in geometry are things, ideas, or objects that demonstrate a certain type of geometry. An application for Euclidean geometry is architecture. Buildings are built with right angles forming squares, rectangles, and other shapes as if they were on a flat plane. An application for spherical geometry is flying in an airplane. When flying in an airplane you follow a geodesic, in order to get to one’s destination in the shortest time possible. An application for hyperbolic geometry is also architecture. Some architecture is being built with straight beams along a curved surface which make the structure look hyperbolic.