Hailey Morris

Mr. Timm & Mr. Mac

STEM 2A

2 October 2014

Differences between Euclidean, Spherical, and Hyperbolic Geometry

The laws of Euclidean geometry and the laws of spherical geometry are very different. While both types of geometry are relevant to the understanding of certain math concepts, they compare in multiple ways. Euclid’s theories were constructed by basing everything on his postulates and definitions, including the Five Postulates. Spherical geometry is only based on four of the Five Postulates and does not coincide with all of Euclid’s theories.

One major different between Euclidean, Spherical, and hyperbolic geometry is the concept of the shortest route from one place to another. In Euclidean geometry, the shortest way to go form one point to another is a line segment. This compares to Spherical and hyperbolic geometry, where the shortest route from one point to another is called a geodesic and is a curved line. Additionally, on a sphere there are no parallel lines; all great circles intersect at two points (great circle is a circle that goes all the way around the sphere). In hyperbolic geometry the parallel postulate also doesn’t work because if you have a line and a point not on that line, you can draw an infinite number of parallel lines (lines look curved but they can always be pulled to show that they are straight lines). This differs from Euclidean geometry because the parallel postulate exists and works. The other Euclidean postulates: there is a straight line from any point to any point, a finite straight line can be produced in any straight line (any line segment can be extended to form a straight line), there is a circle with any center of any radius, and all right angles are equal to one another.

Another major difference between Spherical, Euclidean, and hyperbolic geometry is that involving the sum of the angles for a triangle. In Euclidean geometry, the sum of all the angles of a triangle are always 180 degrees. In comparison, the sum of all the angles of a triangle are always greater than 180 degrees. Also, in spherical geometry the smaller the triangle, the closer the sum of the angles is to reaching exactly 180 degrees, however it never reaches exactly 180 degrees. These two also compare to hyperbolic geometry because again the sum of all of the angle of a triangle are never exactly 180 degrees, however the sum is always less than 180 degrees.

Another difference in Euclidean, Spherical, and hyperbolic geometry deals with similar triangles. In Euclidean geometry, there are similar triangles and congruent triangles. This compares to Spherical geometry where there only congruent triangles, no just similar triangles (if they are similar, then they are congruent). This is because in Spherical geometry, if the angle of the triangle are the same the triangles are the same. This is the same in hyperbolic geometry: if the triangle is similar than they are congruent (no just similar triangles). Another major difference concerning triangles in Euclidean, Spherical, and hyperbolic geometry concerns the Pythagorean Theorem. The Pythagorean Theorem works for Euclidean geometry, but not for spherical or hyperbolic.

Euclidean, Spherical, and hyperbolic geometry also differ because in Spherical geometry there are 2 sided polygons, in hyperbolic geometry is it unsure whether there are 2 sided polygons, and in Euclidean there cannot be a 2 sided polygon. Additionally, for Spherical geometry for a given quadrilateral, the angles will add up to more than 360 degrees. Because a quadrilateral is really made up of 2 triangles and the triangles of Spherical geometry add up to more than 180, the two triangles that make up the triangle will collectively add up to more than 360 degrees. Differing from this, hyperbolic geometry because a quadrilateral is going to add up to less than 360 degrees since the two triangles the quadrilateral are made up of are each less than 180 degrees. Additionally, in hyperbolic geometry there are no rectangles since there is no polygon in which all of the angle are 90 degrees. These both differ from Euclidean geometry because every quadrilateral adds up to exactly 360 degrees since the two triangles that make up the quadrilateral equal exactly 180 degrees.

There are applications for each type of geometry in the world. For Euclidean geometry, some common applications include regular circles, squares, and triangles. Also Euclidean geometry is usually taught in school math classes. Some applications for spherical geometry include a globe, the fact that airplanes move in geodesics around the world to get from one place to another the fastest, and spherical sports balls. Some common applications for hyperbolic geometry include coral and crochet methods in art.

In conclusion, there are many differences in Euclidean, Spherical, and hyperbolic geometry, including things that were not states above. These differences make each type of geometry unique and separate from each other. Spherical, Euclidean, and hyperbolic geometry, however, are all important concepts and can be applied to many situations.