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STEM IIA

23 October 2014

Reflection on Euclidean, Spherical and Hyperbolic Geometry

There are many different applications of Euclidean, spherical, and hyperbolic geometry in our universe. As mathematicians have studies these applications, they have found many differences between the three types of geometry including differences between triangles and other polygons. As spherical and hyperbolic geometry were introduced things like the five postulates began to prove false, therefore these differences began to play a great role in the importance of definitions in geometry.

Euclidean geometry is geometry as looked at on a flat surface. One of the biggest applications of Euclidean geometry is the architecture of most buildings. The flat rectangular shapes of building that we live and interact inside of in everyday life are examples of Euclidean geometry. Although we appear to be living and moving in a Euclidean world, we actually live on a spherical planet. One of the most commonly used applications of spherical geometry is finding the shortest route from one city to another when flying in an airplane. This is called a geodesic. Hyperbolic geometry is not seen as often in everyday life, however, a few applications include coral reefs, lettuce, Pringles and sea slugs. It could also be possible that we live in a hyperbolic universe because it is neither spherical nor Euclidean.

In Euclidean geometry, there are five postulates that are always true. The first four postulates referring to straight lines, circles and angles all also appear to be true in spherical and hyperbolic geometry; however, the fifth postulate does not prove true in spherical and hyperbolic geometry. The fifth postulate states, “Only one line can be drawn through a given point so that the line is parallel to a given line that does not contain the point.” This does not work in spherical geometry because there are no such things as parallel lines in spherical geometry. When drawing two lines (great circles) on a sphere, the lines must cross in two places, therefore there can never be a pair of parallel lines. This also does not work in hyperbolic geometry because an infinite amount of lines can be drawn through a point that are “parallel” or will never touch the other line. Another difference between hyperbolic, spherical and Euclidean geometry is their rules for triangles and other polygons. In Euclidian geometry, all angles of a triangle must add up to 180º but in spherical geometry, the angles must add up to a sum greater than 180º and less than 540º because of the curve of the surface. This also means that all angles of a square must have a sum greater than 360º because when two congruent triangles are put together they create a square so the sum must be greater than the sum of 180 + 180 (since the sum of the angles in a triangle are greater than 180º.) In hyperbolic geometry, all angles of a triangle add up to less than 180 degrees which differs from both spherical and Euclidean geometry. Lastly, in Euclidean geometry a polygon must have at least three sides, but in spherical geometry a polygon can have 2 sides. This is called a loon which is created by the space between two great circles. This means that in spherical geometry a polygon must have at least 2 sides which differs from Euclidean geometry. It is unknown how many sides a polygon can have in hyperbolic geometry because of its infinite surface meaning we do not know what the surface will be like infinitely.

Because of these differences, definitions became extremely important. If definitions are not clearly stated in the correct geometry, they could be proven incorrect (like the fifth postulate.) The definition for “between” is different in spherical geometry than it is in Euclidian geometry. In Euclidian geometry, if three points are on a line then one is always between the other two; however, in spherical geometry all points are between each other because if one draws the great circle, any point can be in between the other two depending on how one looks at it. The definition of the shortest distance between two points is called a line segment in Euclidean and hyperbolic geometry; however, in spherical geometry, the shortest distance between two points defines a geodesic. A geodesic is created by drawing a great circle through the points and then using the shorter line segment. Clearly, definitions are extremely important because incomplete or insufficient definitions become confusing or can be proven incorrect.