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Euclidian vs. Spherical vs. Hyperbolic Geometry

Euclidian geometry is what is used on earth today and includes five postulates which can be disproven in non-Euclidian geometry. Spherical geometry, a type of non-Euclidean geometry, takes place within other spherical dimensions. Aspects of hyperbolic geometry, which is another form on non-Euclidean geometry, can be found on planes in the real world such as a saddle and on unfamiliar planes such as a poncaré disk. Some differences between Euclidian and non-Euclidian geometries are the angles within triangles, the shape and sides of a polygon and the parallel line postulate. Non-Euclidian geometry changes what people perceive to be true in the real world.

In Euclidian geometry, tringles are a three-sided figure which have three angles that add up to one-hundred eighty degrees. This notion of the sum of the angles being equal to one-hundred eighty is often how one proves if a shape is a triangle and is the only type of triangle. However, in spherical geometry, the sides of a triangle are made up of geodesics since this geometry takes place on a sphere. A geodesic is a curve that connects two points, so it is a spherical equivalent of a line since the triangle lies on a curved surface. If one were to measure each angle of a triangle on a sphere, their sum would be more than one-hundred eighty degrees because the triangle is located on a sphere, which causes the lines to be curved. In fact, it would be impossible to have a triangle with an angle sum of one-hundred eighty degrees or less in spherical geometry because the sides forming a triangle are curved which make the angle sum larger than required in Euclidian geometry.

In addition, other polygons exist in spherical geometry that do not exist in Euclidian geometry because they are formed by geodesics. In order for a shape to be classified as a polygon in Euclidian geometry, it must have at least three sides and angles, and all vertices must be connected by a point. In non-Euclidean geometry, there is such thing as a two-sided polygon which connects at two points. Not enough investigation has been done to prove if there is such a figure in hyperbolic geometry. However, this two sided figure is called a lune in spherical geometry. If one were to plot two points on a sphere and draw a line from each point around the sphere these lines would form two lunes if they cross at a point. The lunes would be upside down and backwards from each other since they are located on opposite sides of a great circle. This figure does not exist in Euclidian geometry because lines are only two dimensional, not geodesic.

The fifth postulate in Euclidian geometry is debatable for many mathematicians because it in invalid in non-Euclidian geometry. The parallel postulate states that if there is a straight line that goes on infinitely, and there is a point off of that line which could be a point on a line parallel to the given line, that if a line were to go through the point it would have to go through the original line too. Again, since spherical geometry takes place on a sphere this statement cannot be supported. On a sphere, there are no parallel lines because geodesic lines are curved therefore the parallel line postulate is not valid on this type of plane.

Similarly, the parallel line postulate does not exist in hyperbolic geometry. There are no lines that can go through a point and a straight line on a hyperbolic plane. Yet, there are an infinite amount of lines that can go through the given point. Hyperbolic geometry does not have one plane which all laws of this geometry are valid. Through these laws, one can find examples of real world objects where some apply. These real world objects include a saddle, a pringle, kale, leaves and coral. Planes that are difficult for the human brain to depict, since there is no accurate visual representation, such as the Poncaré Disk are hyperbolic. A point where seven triangle vertices meet is also hyperbolic. These planes are considered to be hyperbolic not just because the parallel line postulate is invalid, but also because other rules different from spherical geometry apply.

Triangles on a hyperbolic plane are different than one’s in Euclidean and spherical geometry because of the sum of their angles. Due to the curvature of lines in hyperbolic space, angles at a vertex appear to be pinched so they cannot be as large as angles in other geometries. This causes the sum of the angles of a triangle in hyperbolic geometry to be less than one-hundred eighty degrees at all times. It is still possible to have an equilateral or equiangular triangle in hyperbolic space, but the angles will add up to be less than one-hundred eighty degrees. This type of triangle contradicts the qualifications of an equilateral triangle in the Euclidean world. This same curvature that alters triangles on a hyperbolic plane is why rectangles and squares cannot be formed in hyperbolic geometry.

Another way shapes are altered in hyperbolic geometry is when they are on a poncaré disk. This type of disk does not exist in a Euclidean world and cannot be accurately visualized by the human mind. However, this disk is best described as a curved circle whose boundaries are infinite. The shortest distance on a poncaré disk is on the arc of the circle perpendicular to the boundary. The boundaries of this disk are a key component into how shapes are seen on this plane. Objects of the same size can appear like one is larger than the other on a poncaré disk. This is because distances increase as an object gets closer to the open boundary. This causes an object to appear smaller as it nears the boundary even when compared to an object of the same size. This idea is not valid on a sphere because the geodesic curves do not change the size of the shape depending on the location. However, points on a sphere have an antipode on the opposite side.

In conclusion, non-Euclidean geometries change the objectives of Euclidean geometry which is perceived as real world geometry. Spherical geometry takes place on another spherical dimension while hyperbolic geometry aspects can be found both on hyperbolic planes and in the real world. These three geometries all express different views of the parallel line postulate, the formation of triangles and how polygons are visualized. Although some of these planes are easier to understand than others, it is possible to learn the unique components of many different geometries and express them in the real world. This brings up the question of what kind of world exactly do we live on. Our earth is spherical, yet includes Euclidean buildings and teachings in everything we do which are valid locally on a sphere. However, there are also aspects of hyperbolic geometry on earth, and it is possible that the universe itself could in fact be hyperbolic. Overall, Euclidean and non-Euclidean geometries contradict each other and provide an alternate view of what is thought of as real world geometry.