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Euclidean, Spherical and Hyperbolic Geometry

Euclidian, Spherical and hyperbolic geometry all pertain to very different surfaces, thus they share many differences and a few similarities. Euclidian geometry studies geometry on flat surface while spherical geometry studies geometry on spherical surfaces, and lastly hyperbolic geometry studies geometry on various numbers of hyperbolic surfaces. All five of Euclid’s postulates apply to Euclidean geometry while only four of those postulates apply to spherical and hyperbolic geometry. Those four postulates that apply to all three of the geometries are: there is a straight line from any point to another, line segments can form a straight line, all circles have a center and a radius, and lastly all right angles are equal to each other.

An example of the differences between the geometries is the lune. A lune is a two sided polygon that can occur in spherical geometry. Lunes do not exist in Euclidian geometry because it is impossible to have a two sided polygon. When two great circles intersect on a sphere, lunes are created. This property of spherical geometry differentiates with the property of Euclidean geometry that states there cannot be a two sided polygon. It is still unsure if there are loons can be loons on hyperbolic surfaces.

Another example of differences amongst the geometries is the triangle. The sum of the angles in a triangle are always greater than 180° in spherical geometry. The larger the triangle, the larger the excess of the triangle’s angles, in terms of them adding up to 180°. In hyperbolic geometry the sum of a triangles angles is always less than 180°. The larger the triangle on a hyperbolic surface the smaller the sum of its angles. The properties of triangles in spherical and hyperbolic geometry differ with the triangles on flat surfaces because every triangle’s angles in Euclidean geometry add up to exactly 180°.

Furthermore, there is no such thing as parallel lines in spherical geometry, and there are infinite number of parallel lines in hyperbolic geometry. When two line segments are connected entirely around a circle, two great-circles are created. This directly conflicts with Euclid’s parallel postulate. On a spherical surface, two great circles have to intersect in two parts and cannot remain parallel. In hyperbolic geometry, given a line and point not on the line, an infinite number of lines pass through that line which do not interest with the given line.

Likewise the shortest route in spherical geometry is not a straight line, rather it is a geodesic line. A geodesic line is one that is curved on a spherical surface. Similarly on a hyperbolic surface the shortest distance between two points is a line which appears to be curved. This can be modeled in a Poincaré disk.

Both hyperbolic and spherical geometry use the relationship between the surface areas to determine the area of a triangle. And in both spherical and hyperbolic geometry the Pythagorean theory does not work, because the angles of the triangles are skewed. Likewise there are no similar triangles in spherical and hyperbolic geometry because any combination of angles generates a unique triangle with precise side measurements.

These three geometries have many applications. Euclidean geometry applies to the mathematics completed on paper, such as architecture and the most common mathematics taught in school. Spherical geometry can be applied to finding the shortest flight paths for planes, or studying the geometry of our earth, because it is a sphere. Lastly hyperbolic geometry can be applied to shapes found in everyday life, such as coral or kale. Furthermore, it is under question if the universe follows the laws of hyperbolic geometry and in that case we could have a better understanding of the universe and its structure.