Alex Rubin

Stem

Mac/Booker

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Comparing Two-Dimensional Geometries

Euclidean, spherical, and hyperbolic geometry are three different types of two-dimensional geometries. Each type of geometry has similarities and differences that have to do with ideas relating to things such as Euclid’s five postulates, triangles, polygons, definitions, and how they apply to the real world.

One of the ideas that relates to the different types of geometries is Euclid’s five postulates. The first postulate states that there is a straight line from any point to any point and the second postulate states that a finite straight line can be produced in any straight line. The third postulate says that there is a circle with any center and any radius and the fourth says that all right angles are equal to one another. The fifth postulate states that for any point not on a line, there is exactly one line parallel to the given line. All of these postulates are true in Euclidian geometry but only the first four are true for spherical geometry. Since spherical and hyperbolic geometries have curved lines, the first and second postulates apply to them but with curved lines instead of straight lines. For spherical and hyperbolic geometries, there are no line segments but instead have geodesics which are parts of great circles which there are instead of lines. Postulates three and four apply to all three of the geometries because they all have circles with any center or radius and have right angles that are always equal to each other. Postulate five applies to Euclidean geometry because there can be two lines that go on forever but never intersect which means they are parallel but it does not apply for spherical or hyperbolic geometry. The reason the fifth postulate does not apply to spherical geometry is because in spherical geometry there are only great circles which always go through the same point. Since all great circles go through the same point, there can never be parallel lines because the lines with eventually intersect. Therefore, there are no parallel lines in spherical geometry. In hyperbolic geometry, the fifth postulate also does not apply because through a point not on a given straight line, infinitely many lines can be drawn that never meet the given line.

Another concept that relates to the different types of geometries is the characteristics of triangles. Triangles have three sides, angles, and edges and are formed by three line segments or geodesics. There are three different types of triangles which include isosceles, scalene and equilateral. Three non-co-linear points determine a triangle and triangles can have right angles. These concepts of triangles apply to Euclidian, spherical, and hyperbolic geometries. Some characteristics of triangles that differ between geometries include the way area is found and the sum of the angles. For Euclidian geometry, the area of triangles can be found by using the formula, 1/2bh, where *b* is base and *h* is height. For spherical geometry, the area of triangles is found by adding up all of the angles and subtracting by Pi. For hyperbolic geometry, the area is found from Pi subtracted by the sum of the angles. For Euclidean geometry, the sum of the angles is always 180 degrees. For spherical geometry the angle sum is greater than 180 degrees but less than 540 degrees. For hyperbolic geometry, the angle sum is greater than 0 degrees but less than 180 degrees.

Polygons are another figure that have characteristics in which the different types of geometry differ. In Euclidian and hyperbolic geometry, polygons have three or more sides. For spherical geometry, a polygon can have two or more sides. A two-sided polygon in spherical geometry is called a lune which has an area that is a fraction of the area of the sphere. The sides of polygons are made up of great circles in spherical geometry. In spherical and hyperbolic geometries, there are no rectangles because rectangles have for 90 degree angels but if you cut them in half in spherical geometry, the two triangles are greater than 180 degrees or and in hyperbolic geometry, less than 180 degrees which cannot be a rectangle because all four of the angles do not equal 90 degree times four.

Definitions are extremely important when talking about the different types of two-dimensional geometries. The reason that they are so important is that they can be different for each type of geometry. One definition for something in one type of geometry may have a different definition in another type of geometry. For example, in Euclidian geometry, lines can be parallel. This definition is different for spherical geometry because parallel lines do not exist is spherical geometry. It is also different from hyperbolic geometry which states that through a point not on a given straight line, infinitely many lines can be drawn that never meet the given line.

Applications of these geometries can be found in different places. For Euclidian geometry, there are many applications in the world including a building or a car. Since buildings and cars are designed with straight lines, have parallel lines and cars drive in straight lines, they are examples of Euclidian geometry. An example of spherical geometry is a plane. Planes want to fly the shortest distance possible so in order to do that, they have to fly the great circle route. For hyperbolic geometry, an example is the planets’ orbit around the sun. When you are looking at this example, it is an example of hyperbolic geometry but when you are actually there where it is happening, it would seem like it was an example of Euclidian geometry.

Another topic I would like to address is how tessellations relate to each of the types of geometries. For Euclidean geometry, a tessellation is a repeating pattern without space in between that can go on forever. For spherical and hyperbolic geometry, tessellations are repeating patterns that can go on forever but they involve curved lines and geodesics instead of straight lines.

Euclidian, spherical, and hyperbolic geometries have many things in common as well as not in common. The importance of definitions is crucial to the two-dimensional geometries because of the varying definitions for each type of geometry. Through Euclid’s Five Postulates, and the characteristics of triangles and polygons, the differences between each type of geometry are prevalent and their uses often apply and relate to the real world.