Rachel Clemons and Ava Drum

7 Bridges of Konigsberg Notes

History of Leonhard Euler:

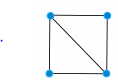
Leonhard Euler was born on April 15, 1707 in Basel Switzerland. He was an academy scholar and worked in the fields of geometry, trigonometry and calculus.

Background on the 7 Bridges of Konigsberg:

The 7 bridges of Konigsberg was based on modern day Kaliningrad, Russia. 7 bridges were built over the river that ran through the city.

The Scheme of the 7 Bridges of Konigsberg:

For centuries mathematicians have pondered Euler's discovery of the relationships between vertices, edges, and degrees, in the 7 bridges of Konigsberg. It is impossible to cross all the seven bridges using a Euler path.

What is a Euler Path? A Euler path is a path that covers every edge of the shape only once. For example, a Euler path is possible for a square with a single line through it. 

(show in video how the pencil will trace each edge)

Your turn: Please use the piece of paper given to you and find which of the following can or cannot have a Euler path. A good way to do this is to trace along each edge without removing your pencil from the paper. Start at one of the vertices (corners). You have 45 seconds.



A B C

Shape A does not have a Euler path. There is no way to trace along each of the edges without retracing at least one edge. (show why this is not possible)

Shapes B and C both have a Euler path. It is difficult. (draw out both paths)

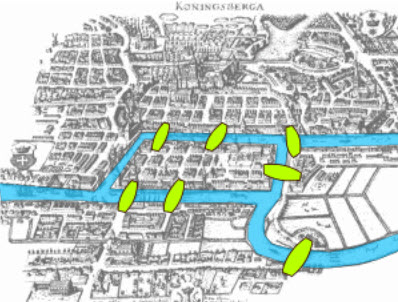
Is there a way to determine if a Euler path can be drawn? Euler helped to figure this out:

Let’s draw some basic diagrams and count the number of vertices in each shape and the number of edges that meet at each vertex, also known as a degree.

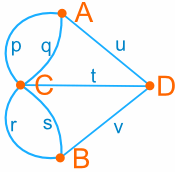
|  |  |  |  |
| --- | --- | --- | --- |
|  | Number of Vertices | Number of Even Degrees | Number of Odd Degrees |
| https://lh3.googleusercontent.com/UezHEidOAZPpYwEdqCkWbgSvoWFbhao13ENgCTxcptVagG5PsOZfXE43M0OiM0LB3M1zIurodcV8PcEllL07WPDZS_sapEmCFDaa4fRjVE_bR4Dw3OSpL_q6Zct4U7KFSh7ddI6W | 4 | 4: (two edges meet at each vertex) (draw this) | 0 (there are no vertices where three or five edges meet) |
| https://lh6.googleusercontent.com/ZOuXRhlKto96IDEU1a8hoB1b2UXrgGjiSASjxgER53z0P06AYeEOoRLrsjq_MGwvmzidEICKpsC7zfV1TJGKaAUM_EqIpp-WhF8BUdnGTfiCzt4V4E7-rsb47XKZJIXS_71RVMA5 | 4 | 2 | 2 |
| https://lh5.googleusercontent.com/QWYH_FbIYncpNzE8pNLHoIHenA_a52s4nNxpKMSd1ZREy105AUieNs4tJlgM08KU-6TLKds6AZijARHF62aOvcZfOVAkzzl-JxTLwrDAeQGJkLRgYSmTOi8vwaynCv0C7gknP7b1 | 4 | 0 | 4 |
| https://lh6.googleusercontent.com/VzXABkJ2PxUiVoh7A2u6HtYjO7eqXybPIHntVmqh5oNWGkwq2dloLVot6r1HkBH04aK8ecj0SeMFMjklFRfbj_LkPlSwxcKfla5I8pC7EI6MIrfv3APp7hWdln-S0bG1xPMrLvY5 | 4 | 2 (top two vertices) | 2 (bottom 2) |
| https://lh4.googleusercontent.com/2GxUGbTazycy5ZMptSvImSNXjykahh9t-5lngTLBjdcBXBr7G_DKQT_j2GTtIPKmrg_Pppy1k6PIOAQXgjvdDBFPXEnD3XB-EOXZZFL7xQ1lumACooNpAyTpcLVl0Uksu4GBlm8t | 6 | 4 (very top 3, middle) | 2 (bottom 2) |

A Euler path works for shapes that have 0 or 2 odd degrees ONLY

This leads back to the Bridges of Konigsberg. The map looks like this



Here is another way the bridges can be drawn



|  |  |  |  |
| --- | --- | --- | --- |
|  | Number of Vertices | Number of Even Degrees | Number of Odd Degrees |
| https://lh6.googleusercontent.com/ZswpHaF-8wBI0-oZpzg93hZ3QNwosR6jEONzN8F5vR5z8OY7ZybSfyV-uetMb2vIB0zwG6v7hNs-bGgHPx79pQa7S68cM4hYQ2pWtIooFR4dWjuO-5aBQYQ2sE8UbMwEE54Dc_i7 | 4 | 0 | 4 |

Therefore, there is no possible way to cross each bridge once because there are 4 odd degrees in the diagram.

The easiest way to make the 7 bridges of Konigsberg possible is to take away the bridge in the middle

Bibliography

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<http://www.mathsisfun.com/activity/seven-bridges-konigsberg.html>

<https://nrich.maths.org/2484>

