Ava Drum

Mr. Timm and Mr. MacDonald

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A Comparison of Euclidian, Spherical, and Hyperbolic Geometries

This paper will compare and contrast three geometries, Euclidian, spherical, and hyperbolic. Euclid was an ancient mathematician who lived around 300 BC in Alexandria Egypt. He is famous for creating the “Elements” consisting of five postulates which are among the most important concepts in geometry. The first four postulates were judged correct by mathematicians that followed Euclid. However, the fifth postulate, the parallel postulate was highly debated. Mathematicians eventually identified two additional geometries that for which the parallel postulate did not work, spherical and hyperbolic geometry.

Applications of each of these geometries can be found in everyday life. An application of Euclidian geometry can be seen in construction of buildings that use many right angles formed by construction materials. Spherical geometry consists of spheres. For example, one important application of spherical geometry can be seen in air plane travel. When you travel from Florida to the Philippines, the shortest distance is by following the great circle (described later) over Alaska, even though the Philippines is to the south of Florida. An application of hyperbolic geometry is the saddle shaped (hyperbolic paraboloid) Pringles potato chip, a shape chosen because of the higher chip strength provided by the shape resulting in a lower level of chip breakage.

Definitions are important when describing differences among Euclidian, spherical, and hyperbolic geometry. For example, in Euclidian geometry a line is defined as a straight line on a flat surface. In spherical geometry a line is an arc and is not on a flat surface. This difference in the definition of a line determines whether Euclid’s postulates are right or wrong.

Euclidian’s first postulate states that there is a straight line from any point to any point. This postulate works as well with spherical and hyperbolic geometry when you replace a straight line with a curved geodesic lines. An example of a geodesic line would be a longitudinal line extending between the north and south poles on the earth. Euclidian’s second postulate states that a straight line segment can be extended indefinitely in any straight line. This postulate works in spherical geometry because geodesic lines form great circles which represent the largest circle that can be drawn on any given sphere, and these lines can go around the sphere an infinite amount of times. In hyperbolic geometry, a line can be extended indefinitely. Euclidian’s third postulate states that there is a circle with any center and any radius. This postulate works in spherical geometry because every sphere has a center and a radius. However, there is a limit to the length of the radius as defined by the size of the sphere. The postulate works in hyperbolic geometry; however, the center is not located where a Euclidian center would be. Euclidian’s forth states that all right angles are equal to one another. In Euclidian geometry, right angles are formed by two intersecting perpendicular straight lines. This is true for spherical geometry because the geodesic lines can intersect and create perpendicular lines (right angles). Right angles in hyperbolic geometry are also congruent. Euclidian’s fifth postulate states for a line and a point not on a line, there is only one line containing a point that is parallel to the original line. As mentioned earlier, this is Euclid’s famous parallel postulate. This postulate does not hold for spherical geometry because there are no parallel lines as illustrated by two of the earth’s longitudinal lines stretching between the poles. They can never be parallel. For hyperbolic geometry, the postulate does not hold because there are an infinite number of parallel lines through the point. There can only be one parallel line for this postulate to be true.

Triangles provide an interesting way to compare Euclidian, spherical, and hyperbolic geometries. In Euclidian geometry, a triangle is formed by three straight lines on a flat surface. In Spherical geometry, triangles are formed by three great circles and have arced lines and are not on a flat surface. There are no similar triangles in hyperbolic geometry; however, there are similar triangles in Euclidian geometry. The sum of the angles of a Euclidian triangle is 180 degrees. In spherical geometry, the sum of the angles of a triangle can be more than 180 degrees. In hyperbolic geometry, the sum of the angles in a triangle are always less than 180 degrees. In Euclidian geometry, a triangle cannot have more than one right angle. In spherical geometry, a triangle can have more than one right angle as illustrated by constructing two geodesic curves 90 degrees apart stretching from the North Pole to the equator. The shape of this triangle has three 90 degree angles.

With respect to polygons, there are differences as well among Euclidean, spherical and hyperbolic geometries. In Euclidian geometry, a polygon must be drawn on a flat plain with three or more lines. In spherical geometry, a polygon can be drawn with as few as two lines or more. As illustrated again by a pair of longitudinal lines between the north and south poles of the earth. When two great circles intersect a polygon is created. This is because all great circles intersect creating a polygon. The angles of a quadrilateral are greater than 360 degrees. There are no squares or rectangles in hyperbolic space because the sum of the angles of a quadrilateral has to be less than 360 degrees.

In conclusion, Euclidian, spherical, and hyperbolic geometries provide a different way of looking at the world which forces people to see things differently and explore visual concepts that are not obvious. Tessellations provide a way for Euclidian and non-Euclidian geometry come alive. It can be illustrated in art, architecture, and tiling. Tessellations provide a great way to look at these geometries in a different way. There are tessellations illustrated below which are some of my favorites, including bats and angles hyperbolic tessellation by M. C. Escher.

