STEM

10/22/15

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The Five Postulates: Euclidian, Spherical, & Hyperbolic Geometry

The Five Postulates were made by Euclid of Alexandria, a mathematician often referred to as the “Father of Geometry.” These postulates are common notions that occur in Euclidian geometry. They are as follows: “A straight line segment can be drawn joining any two points, any straight line segment can be extended indefinitely in a straight line,” “Given any straight lines segment, a circle can be drawn having the segment as radius and one endpoint as center,” “All Right Angles are congruent,” and “If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two Right Angles, then the two lines inevitably must intersect each other on that side if extended far enough.” Definitions are necessary because that way people have a common name and understanding for what that name stands for or represents. Here the definitions of concepts and shapes we have known our entire lives will change.

Different postulates either apply or don’t apply to different geometries. The same postulate can apply to different geometries in different, valid ways. In the first quarter of STEM IIB, we have learned about the postulates and if and how they apply to three types of geometries: Euclidian, Spherical, and Hyperbolic. They apply differently to the different geometries because of the differences in the geometries’ structure, and therefore differences in nature and truth. 🡪 ADD OR NOT? DEFINE HERE OR LATER? Euclidian geometry is the type of geometry that exists in our world. Spherical geometry is like a ball, where the world is spherical. Hyperbolic geometry is best represented by a piece of kale, or other leaf. In Hyperbolic geometry, there are valleys and craters. Here, depending on which parts you stretch and what parts you leave scrunched up, different things are true.

Euclidian Geometry is the most familiar geometry, it has been taught to us since we were young.

Euclidian Geometry is the geometry with which we are most familiar; it was the first type we came in contact with as children. It is the geometry in the material world we are exposed to: flat surfaces. Here, it is possible to make straight lines, four ninety degree angles, and perfectly round circles. In this type of geometry, the first postulate is true: a straight line segment can be drawn between two points. This is because it is on a flat surface, the line can take the shortest path (a straight line) between the two points. The second and third postulates are also applicable: a straight line can start at one point and go on in one direction forever (this is because the flat surface goes on forever, and the line is straight, so it would travel indefinitely in the same direction,) and a circle can be made using a segment as a radius and turning it, which can be done here because landscape is flat. Right angles are all congruent, and on a squares and rectangles each have four, but triangles can only have one right angle, proving the fourth postulate right in this geometry. The angles of triangles add up to 180 degrees. As for the fifth postulate, in Euclidian Space for every point not on a line there is one line parallel to the given line.

The next type of geometry we will inspect is Spherical Geometry. Spherical Geometry is when the surface is not a plane, but a rounded, like a ball or sphere. The first postulate is accurate: there can be two lines drawn on the sphere, called Geododes, one going each way of the sphere, connecting the two points. When a line comes full circle around the following place of largest circumference, it is called a Great Circle. There cannot be a straight line extended indefinitely in a straight line; the line will eventually find itself in its origin because the line is on a sphere, making the second postulate not true in this type of geometry. Like in Euclidian Geometry, there can be a circle having a segment as a radius and one endpoint as a center. The fourth postulate is true, right angles can be congruent, but they will be more than ninety degrees because they are drawn on a rounded surface. Unlike the Euclidian Geometry, even triangles can have multiple ninety degree angles. Finally, for every point not on a line, there is not a line parallel to the given line. This is because it is on a rounded surface, and the lines would be the same sides, geododes, and eventually intersect at two points.

The final Geometry we learned about was Hyperbolic Geometry. It is like a saddle, curved in two ways, and very large. Here, like in Euclidian Geometry, you can draw one line between two points, although it is not always straight. Sometimes, it is a shorter distance to go around the mountain than up and back down it. Unlike the other geometries, there cannot be a circle made using a straight segment as its radius. This is because the field has “wrinkles” in the skin, with valleys and mountains. This makes it impossible to make a true circle, because the surface is too wavy for that, making the third postulate inaccurate. Right angles can also not be congruent, because when you flatten out one part, the other right angle changes so it is either bigger or smaller than it was before, and not equal. This means that the fourth postulate is false in this geometry. For every point not on the line, there are multiple lines parallel to the given line. This is because different lines are parallel depending on what parts are flat and what parts are wrinkled. Like in spherical geometry, here the fifth postulate is not true.

The five postulates are very useful in highlighting the similarities and differences between different geometries. The five postulates were discussed in the different worlds of Euclidian, Spherical, and Hyperbolic Geometry. Each had an individual set of features that were either in agreement or denied the postulates. This helped us see how the different geometries were the same in some respects and different in other ways. If I were to look into a new topic, it would be a spiral on a sphere. This is because it can grow larger and smaller depending on how much it is compacted.