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Euclidean vs. Non- Euclidean Geometry

Euclidean Geometry can be described simply as the study of plane and solid figures based on axioms and theorems employed by the Greek mathematician Euclid. This is the geometry most commonly taught in secondary schools worldwide. Euclid lived in the time of 300 BCE and was the most prominent mathematician of the Greco-Roman time, best known for his written work, the *Elements*. Euclid came up with the 5 postulates: (1.) There is a straight line from any point to any point (2.) A finite straight line can be produced in any straight line (3.) There is a circle with any center and any radius (4.) All right angles are equal to one another and lastly (5.) If a straight line falling on two straight line makes the interior angles on the same side less than two right angles. These postulates were all assumed and because of this, many other mathematicians, such as C. F. Gauss, N. Lobachevsky, J. Bolyai and B. Riemann, proved the 5th postulate to be untrue, discovering both Non-Euclidean geometries – Spherical and Hyperbolic. Spherical geometry is impacted by the 5th postulate due to the fact that there are no parallel lines, while the 5th postulate in Hyperbolic geometry does not work because there are infinite many lines that go through a given point.

With the discovery of Non-Euclidean geometry, the 5th postulate enabled each geometry to have different properties. In Euclidean geometry, a straight line is the shortest way to get from one point to another. In spherical geometry, this statement is seen to be untrue, as the shortest way to get from one point to another is by a geodesic. A geodesic is a curve that traces the shortest distance between any two of its points. Every geodesic is considered a great circle. Polygons differ in the two geometries as well. Euclidean geometry has a minimum of three sides while the curviness of a geodesic enables a polygon to have a minimum of two side in spherical geometry. This two-sided shape is called a lune. Mathematician, Albert Girard, came up with a way to calculate the surface area of a lune of a sphere. This procedure is called Girard’s Theorem. Unfortunately, there are no polygons in hyperbolic geometry.

Other shapes, such as triangles and circles, differ in all three geometries. In Euclidean geometry, the angles of a triangle will always add up to 180 degrees. In spherical geometry, the sum of a small triangle must be more than 180 degrees, and the largest triangles will always be less than 900 degrees. As the area increases in a triangle, the angle sum needs to increase as well, proving that because of Girard’s theorem, it is impossible for two triangles to be similar without being congruent. In hyperbolic geometry, every triangle will be less than 180 degrees since all lines appear to be curved until one places their fingers on two points, pulls, and discovers that the line will then become straight. As for circles, every geometry but Euclidean is more complex than just a ‘regular’ circle drawn on a piece of paper. In spherical geometry, all circles surround a sphere. As the radius increases from the top of the sphere, the circle increase until it reaches the great circle and then decreases its radius once more. A great circle can be defined as a circle on the sphere cut out by intersecting the sphere with a plane through its center. As for hyperbolic geometry, no circles can be created.

When comparing and contrasting Euclidean and Non-Euclidean geometry, definitions of properties and structures are greatly important, and by defining such properties, one could begin to think of how all three geometries serve importance in everyday life. Each geometry depends tremendously on definitions. For example, a polygon in Euclidean geometry is a shape with a minimum of three sides while a polygon in spherical has a minimum of two sides. Definitions serve the purpose of enabling the learner to be able to differentiate similar words and understand their purpose in all three geometries. With these distinct definitions, one can explore their surroundings and easily discover everyday applications. Many structures are built by applying Euclidean geometry, such as paintings and arts, building structures, etc. To make the decorations contributed in art for weaving, pottery, and other objects, early artists experimented with symmetries and repeating patterns of 3 sided polygons. As for architecture, squares and straight lines are needed for the structure to have a strong foundation and stay intact. For spherical, pilots and sailors use this geometry to map out the shortest distance for travel. Since the Earth has a spherical form, Euclidean rules cannot be applied. Lastly, hyperbolic geometry can be displayed through the path sound makes that is produced by a whale in which they take as their shortest route of travel. Sound does not travel at a constant speed and below a certain depth, it travels at a speed that is measured the same as the depth under the surface. This lack of constant speed creates a curved path. These scenarios are just a few out of the many applications in this world we can think of!

In conclusion, Non-Euclidean geometry was derived from the possibility that anything is possible if you stick your mind to it. With Euclid creating his 5 postulates out of assumption, it differed all three geometries and allowed inclined judgement. All three geometries have influenced a new way of learning, enabling researchers think deeply and possibly discover something worthwhile.